Comment on "21-cm Radiation: A New Probe of Variation in the Fine-Structure Constant"

(Dated: May 20, 2010)

Khatri and Wandelt reported that change in the value of α by 1% changes the mean brightness temperature T_b decrement of the CMB due to 21 cm absorption by 5% over the redshift range z < 50. A drawback of their approach is that the dimensionful parameters are used. Changing of units leads to the change of the magnitude and even sign of the effect. Similar problems may be identified in a large number of other publications which consider limits on the variation of α using dimensionful parameters. We propose a method to obtain consistent results and provide an estimate of the effect.

In the Letter [1] Khatri and Wandelt investigated the effect of variation in the value of the fine-structure constant α at high redshifts and reported that change in the value of α by 1% changes the mean brightness temperature T_b decrement of the CMB due to 21 cm absorption by 5% over the redshift range z < 50. The authors made this conclusion starting from expressions

$$T_b = \frac{(T_s - T_\gamma)\tau}{1+z}, \qquad \tau \equiv \frac{3c^3\hbar A_{10} n_H}{16 k_B \nu^2 H T_s}, \tag{1}$$

where T_s is the spin temperature, T_γ is the radiation temperature given by $T_\gamma \approx 2.73(1+z)$K, z is the redshift, c is the speed of light, k_B is the Boltzmann constant, ν and A_{10} are the frequency and the probability of the hyperfine $(1S_{1/2}, F=1)$ – $(1S_{1/2}, F=0)$ transition in hydrogen; n_H is the total number density of hydrogen nuclei, and H is the Hubble parameter at redshift z. Based on the following estimates $\nu \sim \alpha^4$, $A_{10} \sim \alpha^{13}$, and using Eq. (1) the authors of Ref. [1] come to conclusion that $T_b \sim A_{10}/\nu^2 \sim \alpha^5$ giving $\Delta T_b/T_b = 5\%$ for 1% change in α.

A drawback of this approach is that the dimensionful parameters are used. Up to a numerical constant, the frequency
$\nu \sim \alpha^4 g_I (m/m_p)(mc^2/h)$,
where m is the electron mass, m_p is the proton mass and g_I is the proton magnetic g factor. Thus, in units mc^2 we have $\nu \sim \alpha^4$. In the atomic units (which are more natural for the atomic problem we deal with) $\nu \sim \alpha^2$. If the frequency is measured in conventional SI units s^{-1} (defined using the Cs atom hyperfine frequency) $\nu \sim \alpha^{-0.83}$ [2], i.e. the effect has an opposite sign.

Moreover, even in units of mc^2 used by the authors of [1] we can obtain a different result. The temperature T_b is actually defined using the observed intensity $I_\nu = 2k_B\nu^2 T_b/c^2 \sim \alpha^{13}$. Therefore, $\Delta I_\nu/I_\nu = 13\%$ for 1% change in α.

Similar problems may be identified in a large number of other publications which consider limits on the variation of α using dimensionful parameters. Simple replacement of the electric charge squared by α in all equations and its variation gives meaningless results.

Such problems do not appear in the laboratory (atomic clocks) and quasar absorption spectra measurements of the variation of the fundamental constants. From the very beginning these studies deal with the dimensionless ratios of the atomic transition frequencies [2–4]. The atomic unit of energy $me^4/\hbar^2$ cancels out in the ratios of the frequencies. The dependence on $\alpha = e^2/\hbar c$ appears from the dimensionless ratio of the relativistic corrections to the atomic unit of energy.

In the quasar spectra analysis [4] many frequencies are used to find α variation and redshift. The redshift cancels out in the ratios of the frequencies. Therefore, α variation is determined by these ratios.

Let us try a similar approach of using dimensionless ratios for the variation of the brightness temperature. It is reasonable to start from the ratio T_b/T_γ which is given by (see Eq. (1))

$$X_T \equiv \frac{T_b}{T_\gamma} = \frac{(1 - T_\gamma/T_s)}{1+z}\frac{3c^3\hbar A_{10} n_H}{16 k_B \nu^2 H T_\gamma}. \tag{2}$$

The dimensionless atomic parameter here is
$X_A \equiv A_{10}/\nu \sim \alpha^9 g_I^2 (m/m_p)^2$.
Fortunately, Eq. (1) from [1] which determines the ratio T_γ/T_s, depends on the same atomic parameter X_A.

The remaining dimensionless parameter
$X_H = (c^3\hbar n_H)/(k_B \nu H T_\gamma)$
contains the hydrogen density $n_H = \eta n_\gamma \sim \eta T_\gamma^3$ where n_γ is the photon density. The numerical value of the proton-to-photon number ratio η has been obtained from CMB (or BBN) data in assumption that there was no variation of the fundamental constants. If there was any variation, this value would be different (for example, the equation for the ionization equilibrium contains combination $\eta\alpha^3$). Therefore, X_H is actually sensitive to the value of α at CMB era where $\delta\alpha$ could be larger. We leave this complicated problem for a future study. Note that in the case of BBN we calculated how the best fit of η is affected by the change of the fundamental constants [5]. Now it seems reasonable to assume that the dependence of X_H on the fundamental constants is weaker than the dependence of X_A which is enhanced by an order of magnitude by the factor α^9. Then we obtain
$\delta X_T/X_T \sim 9\delta X_\alpha/X_\alpha$,
where $X_\alpha = \alpha[g_I(m/m_p)]^{2/9}$. The dependence of the proton g-factor and m/m_p on the fundamental constants has been presented in Ref. [2]: $g_I \sim (m_q/\Lambda_{QCD})^{-0.1}$, and $m/m_p \sim (m/\Lambda_{QCD})(m_q/\Lambda_{QCD})^{-0.05}$ where m_q is the quark mass and Λ_{QCD} is the strong interaction QCD scale.

A rough estimate of the redshift dependence of the X_α variation effect may be extracted from the z-dependence

of the α variation effect presented on Fig. 2 of Ref. [1]. Indeed, according to the estimates given above the relative difference in these effects is $\sim 9/5$.

In principle, one may try to reduce the problem to that calculated in Ref. [1] by considering (instead of X_A) a different dimensionless parameter
$X_B \equiv A_{10} T_\gamma/\nu^2 \sim \alpha^5 g_I (T_\gamma/m_p c^2)$.
A minor problem here is that the equation for the spin temperature T_s presented in Ref. [1] contains X_A (rather than X_B). A major problem is how to reduce the variation of the ratio $T_\gamma/m_p c^2$ to the variation of the dimensionless fundamental constants. The CMB temperature is the red-shift-dependent phenomenological parameter which depends on units we use, and these units may be time-dependent. Even if the variation of X_B is "detected", it would be hard to provide an interpretation in terms of theories of the fundamental constants variation.

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